



Maths Workshop

November 2019

KS1 Year 2

What we will cover

Explanation of:

- Different methods and strategies used
- Concrete-Pictorial-Abstract approach
- Bar modelling
- Calculation & problem solving examples

Levels of learning

Shallow learning: surface, temporary, often lost

Deep learning: it sticks, can be recalled and used

Deepest learning: can be transferred and applied in different context

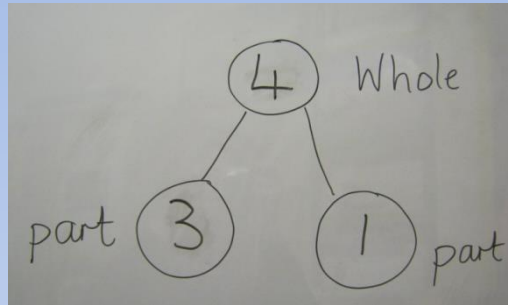
What is mastery?

- The essential idea behind the mastery teaching approach is that ***all pupils*** gain a deep understanding of the mathematics. This ensures that:
- *Future* mathematical learning is built on *solid foundations* which do not need to be re-taught (less breadth but greater depth)
- Increasingly, there will be less *need* for separate catch-up programmes due to some children falling behind;
- Pupils who, under other teaching approaches, can often fall a long way behind, are better able to keep up with their peers, so that gaps in attainment are narrowed whilst the attainment of all is raised.

Concrete

Pictorial

Abstract



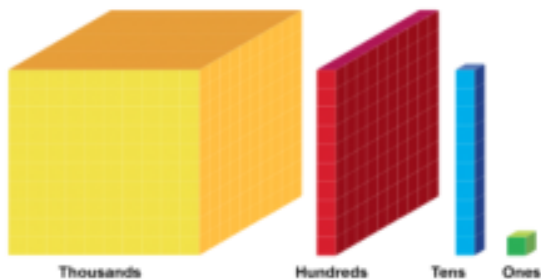
$$3 + 1 = 4$$





Concrete or pictorial representations support students to understand abstract concepts

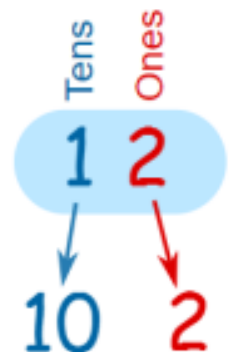
Place Value...

Place value is the value of each digit in a number.
It means understanding that 82 is made 80 and 2,
rather than 8 and 2.



Tens	Ones
1	2
	

The Number "12"



Place Value...

Place Value...

Match the representation to the correct number.



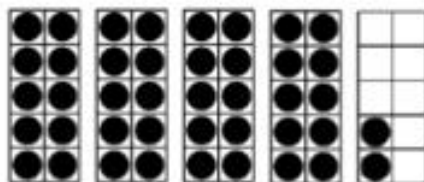
•

• forty-one



•

• 24



•

• 4 tens and 2 ones

Place Value...

Complete the part whole models.

place value

2

What part do we know?

How can we use the whole and part to work out the missing part?

Place Value...

It is important that children can partition numbers in a variety of ways, not just as tens and ones. For example, 58 is made up of 5 tens and 8 ones or 4 tens and 18 ones, or 20 and 38, etc.

Complete the part whole models.

place value

2

What part do we know?

How can we use the whole and part to work out the missing part?

Place Value...

How many two-digit numbers can you make using the digit cards only once?



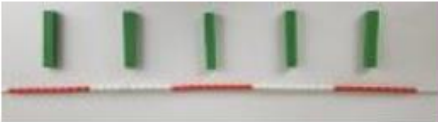

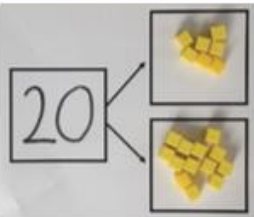
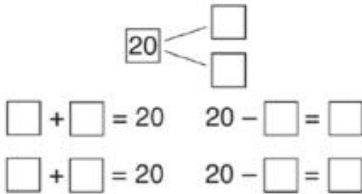
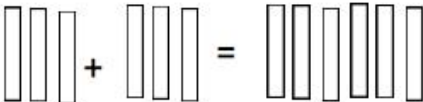
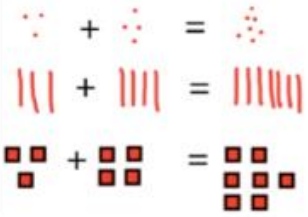


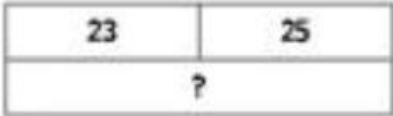
I can make _____ two-digit numbers.

They are _____

Your turn!

Numeracy key words

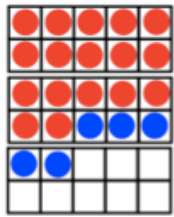
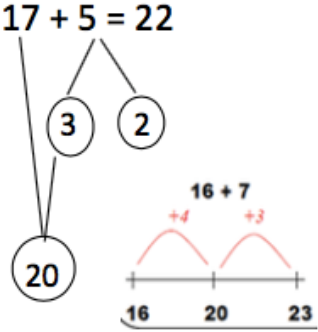
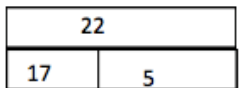

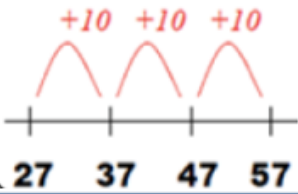
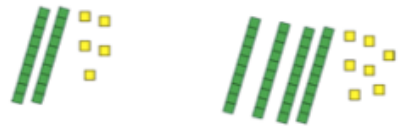
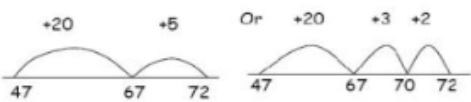

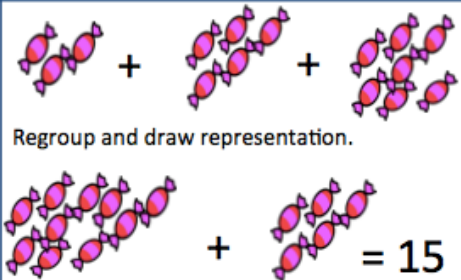
Addition +	Subtraction -	Multiplication X	Division ÷
More than	Subtract	Multiply	Divide by
Total	Minus	Lots of	Share
Altogether	Less than	Times	Groups of
Plus	Take away	Multiplied by	
Add	Difference between	Multiples of	

Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten	$50 = 30 + 20$  Model using dienes and bead strings	 $3 \text{ tens} + 5 \text{ tens} = \underline{\hspace{2cm}} \text{ tens}$ $30 + 50 = \underline{\hspace{2cm}}$ Use representations for base ten.	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$
Use known number facts <i>Part part whole</i>	 Children explore ways of making numbers within 20	 $\square + \square = 20$ $20 - \square = \square$ $\square + \square = 20$ $20 - \square = \square$	$\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$
Using known facts	$\square\square + \square\square = \square\square\square\square$ 	 Children draw representations of H,T and O	$3 + 4 = 7$ <i>leads to</i> $30 + 40 = 70$ <i>leads to</i> $300 + 400 = 700$
Bar model	 $3 + 4 = 7$	 $7 + 3 = 10$	 $23 + 25 = 48$

Y2

15

ADDITION +

Objective & Strategy	Concrete	Pictorial	Abstract
Add a two digit number and ones Children explore the pattern. $17 + 5 = 22$ $27 + 5 = 32$	 $17 + 5 = 22$ Use ten frame to make 'magic ten'	$17 + 5 = 22$ Use part part whole and number line to model. 	$17 + 5 = 22$ Explore related facts $17 + 5 = 22$ $5 + 17 = 22$ $22 - 17 = 5$ $22 - 5 = 17$ 
Add a 2 digit number and tens $25 + 10 = 35$ Explore that the ones digit does not change	 $25 + 10 = 35$	$27 + 30$ 	$27 + 10 = 37$ $27 + 20 = 47$ $27 + \square = 57$
Add two 2-digit numbers Model using dienes, place value counters and numicon		 $47 + 25 = 72$ Use number line and bridge ten using part whole if necessary.	$25 + 47$ $20 + 5$ $40 + 7$ $20 + 40 = 60$ $5 + 7 = 12$ $60 + 12 = 72$
Add three 1-digit numbers Combine to make 10 first if possible, or bridge 10 then add third digit		 $47 + 6 = 53$ $53 + 2 = 55$ $47 + 6 + 2 = 55$	$4 + 7 + 6 = 10 + 7 = 17$ Combine the two numbers that make/ bridge ten then add on the third.

Addition

$$23+12 =$$

Using a number line...



Addition

Using partitioning...

+

Partitioning both numbers into tens and ones mirrors the column method where ones are placed under ones and tens under tens. This also links to mental methods.

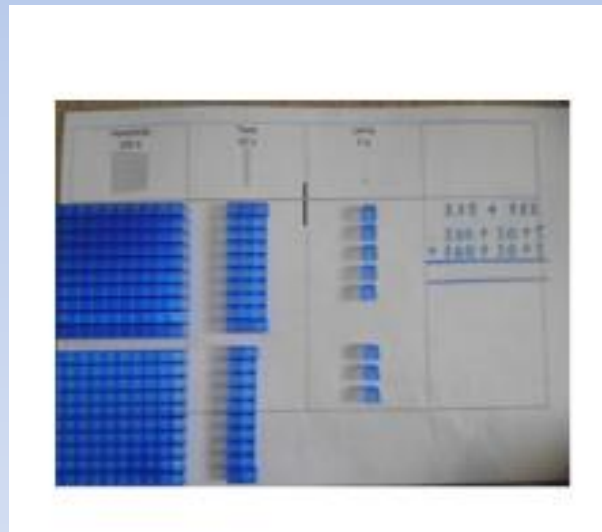
The expanded method **leads** children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.

$$76 + 47$$

$$70 + 40 = 110$$

$$6 + 7 = 13$$

$$110 + 13 = 123$$



+

$$\begin{array}{r} 39 \\ 22 \\ \hline 11 \\ 50 \\ \hline 61 \end{array}$$

Addition

Using columns...

+

In this method, recording is reduced further. Carry digits are recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.

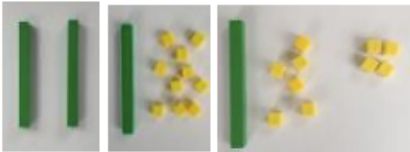
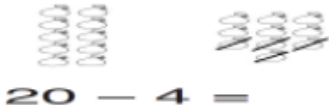


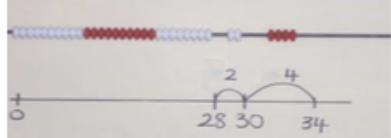
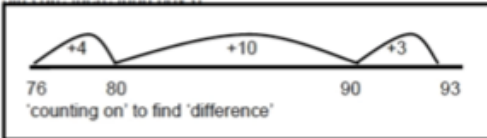
$$\begin{array}{r} 326 \\ + 254 \\ \hline 580 \end{array}$$

The bar model is a really good way of helping children to understand the relative sizes of numbers and to link three numbers together in different ways, showing addition and subtraction are closely related (inverse).

There are 20 sweets in my bag and 13 sweets in my friend's bag. How many sweets have we got altogether?	
20	13

Number families...



Objective & Strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 <p>Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'</p>		$20 - 4 = 16$
Partitioning to subtract without regrouping. 'Friendly numbers'	$34 - 13 = 21$  <p>Use Dienes to show how to partition the number when subtracting without regrouping.</p>	<p>Children draw representations of Dienes and cross off.</p>  $43 - 21 = 22$	$43 - 21 = 22$
Make ten strategy <i>Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</i>	 $34 - 28$ <p>Use a bead bar or bead strings to model counting to next ten and the rest.</p>	 <p>Use a number line to count on to next ten and then the rest.</p>	$93 - 76 = 17$

Y2 SUBTRACTION-

Subtraction

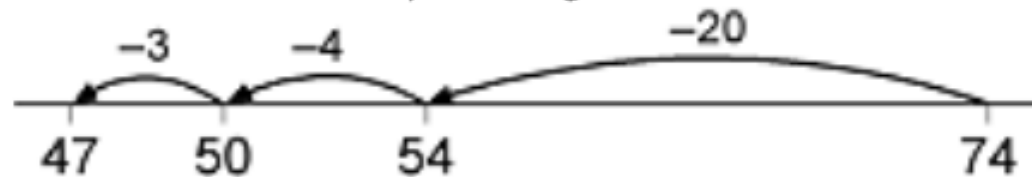


Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10.

$$15 - 7 = 8$$



$74 - 27 = 47$ worked by counting back:



Subtraction

Using a number line...



$$28 - 13 =$$



Subtraction

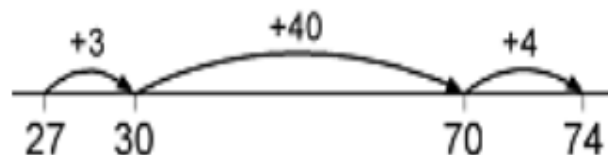
Finding an answer by counting up - The steps can also be recorded by counting up from the smaller

to the larger number to find the difference, for example by counting up from 27 to 74

in steps totaling 47. Pupils should be comfortable that either way we get the same answer

(and that the three numbers involved therefore hold an inverse relationship).

$$74 - 27 =$$



Subtraction



Expanded layout, leading to column method

- Partitioning the numbers into tens and ones and writing one under the other mirrors the column method, where ones are placed under ones and tens under tens.
- The expanded method leads children to the more compact method so that they understand its structure and efficiency.

Example: $63 - 41 =$

$$\begin{array}{r} 60 + 3 \\ - 40 + 1 \\ \hline + \\ \hline \end{array}$$

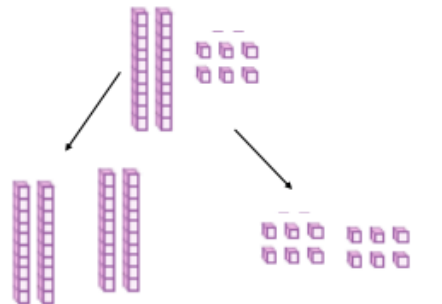
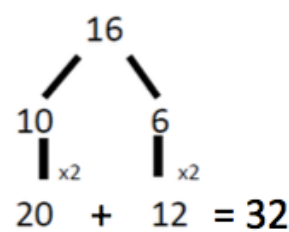




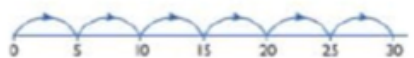

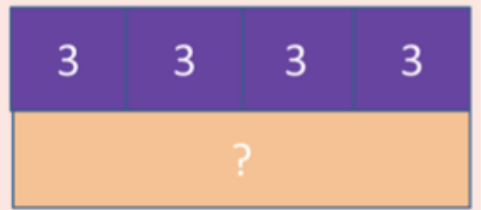
Start by subtracting the ones, then the tens. Refer to subtracting the tens, for example, by saying 'sixty take away forty', not 'six take away four'.

Subtraction



The concept of transfer / exchange

$$\begin{array}{r} 72 \\ - 47 \\ \hline \\ \hline \end{array}$$

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling	<p>Model doubling using dienes and PV counters.</p>  $40 + 12 = 52$	<p>Draw pictures and representations to show how to double numbers</p>	<p>Partition a number and then double each part before recombining it back together.</p>  $16 \begin{matrix} \swarrow & \searrow \\ 10 & 6 \end{matrix} \begin{matrix} \times 2 & \times 2 \\ 20 & 12 \end{matrix} \rightarrow 20 + 12 = 32$
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.</p>  $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$  	<p>Number lines, counting sticks and bar models should be used to show representation of counting in multiples.</p>    	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10</p> <p>0, 3, 6, 9, 12, 15</p> <p>0, 5, 10, 15, 20, 25, 30</p> $4 \times 3 = \square$

Multiplication

We started with step counting in Year 1 and still do this but we also look for patterns and start to know that $1 \times 2 = 2$ and $2 \times 2 = 4$ etc.

Which times table has been highlighted on the number square?

How do you know?


Which of these numbers would you find in the 5 times table?

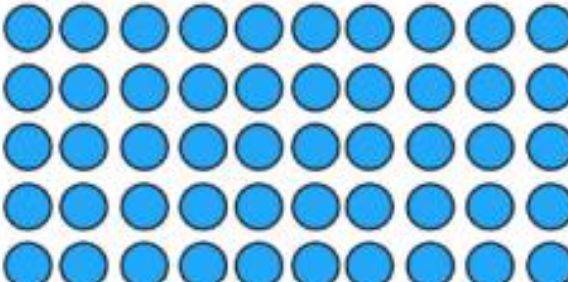
134 **67** **205** **502**

How do you know?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Multiplication

$$2 \times 10$$


$$5 \times 10$$


Arrays



Multiplication



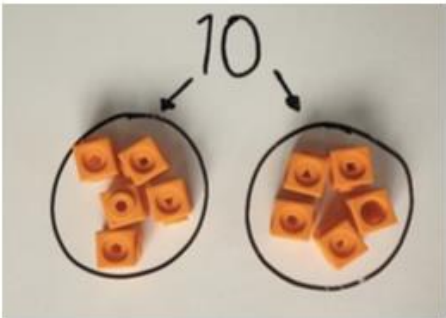

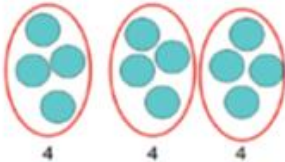
$$3 \times 3 =$$

$$3+3+3=$$

Using a number line

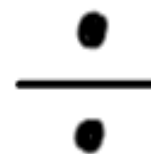


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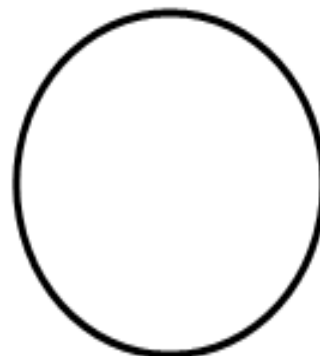
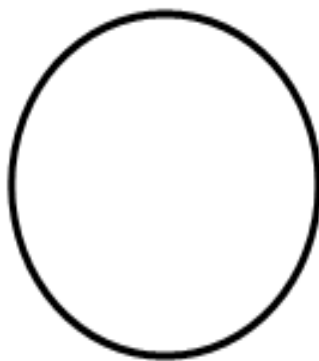
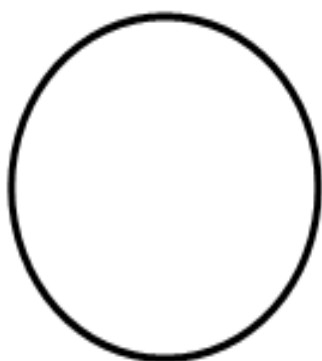
Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing <i>Use Gordon ITPs for modelling</i>	   <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities.</p>  <p>8 shared between 2 is 4</p> <p>Sharing:</p>  <p>12 shared between 3 is 4</p>	<p>12 shared between 3 is</p> <p>4</p>

Division

By sharing



The farmer had 12 sheep. He put them into three fields. How many sheep were in each field.



$$12 \div 3 =$$



Division

$12 \div 3 =$

Using a number line \div
and repeated subtraction

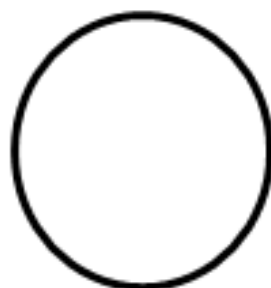
12

Mental starters - brain warmers!

Prove it!! (Explain and justify)

Which is the odd one out?

4 5 8



Mental starters - brain warmers!

Missing number problems...

$$14 + \underline{\quad} = 23$$

$$25 - \underline{\quad} = 20$$



Mental starters - brain warmers!

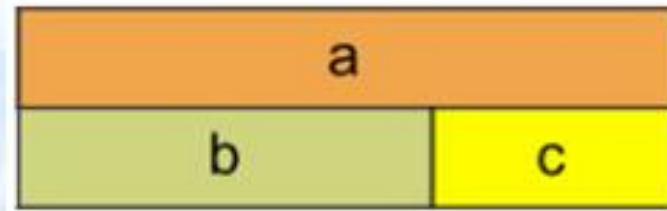
$$20 + 14 = 30 + 4$$

(is the same as)

$$30 + 11 = 20 + \underline{\quad}$$



Bar model – pictorial representation of a problem



Using this model children can understand these relationships:

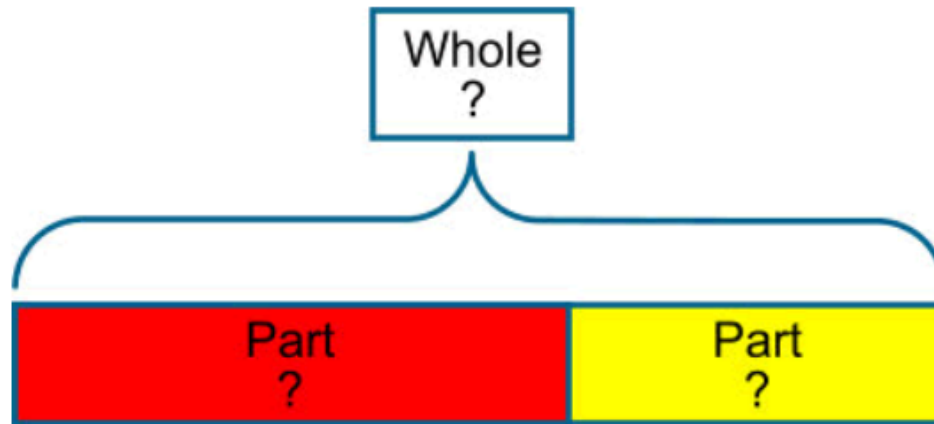
$$b + c = a$$

$$c + b = a$$

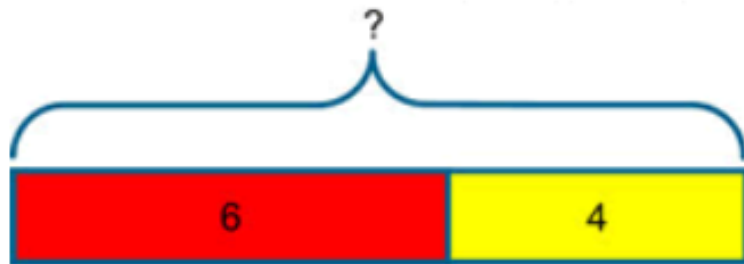
$$a - b = c$$

$$a - c = b$$

In problems involving addition and subtraction there are three possible unknowns as illustrated below and given the value of two of them the third can be found.



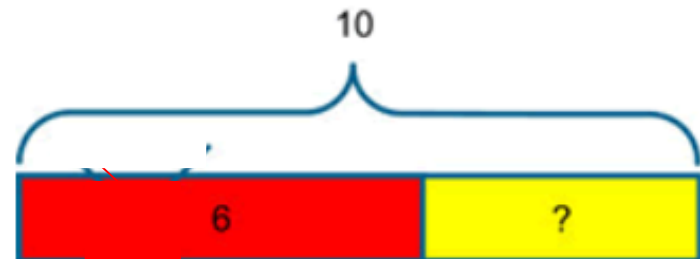
Addition



I have 6 red pencils and 4 yellow pencils. How many pencils do I have?

(I combine two quantities to form the whole)

**Subtraction
- Take Away**



I had 10 pencils and I gave 6 away, how many do I have now?

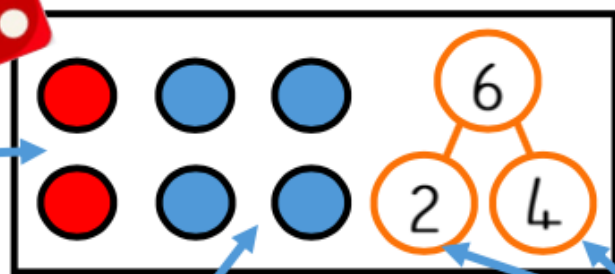
(This time we know the whole but only one of the parts, so the whole is partitioned and one of the parts removed to identify the missing part)

Make 6

1. Roll dice

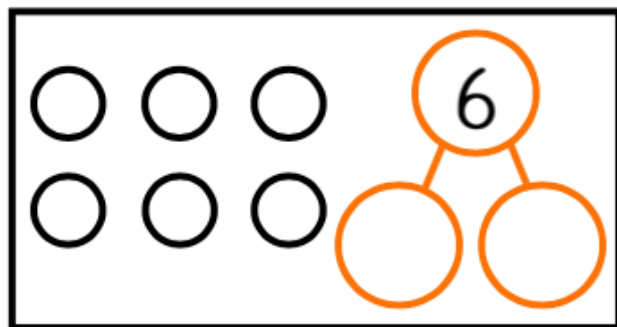
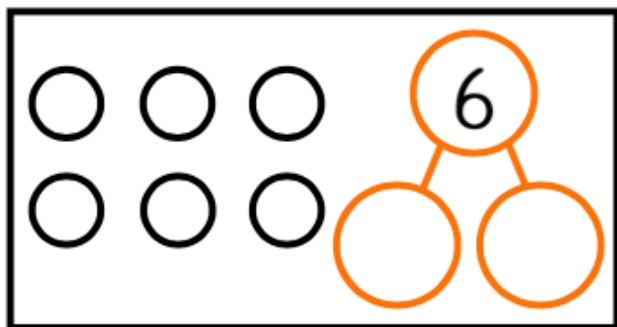
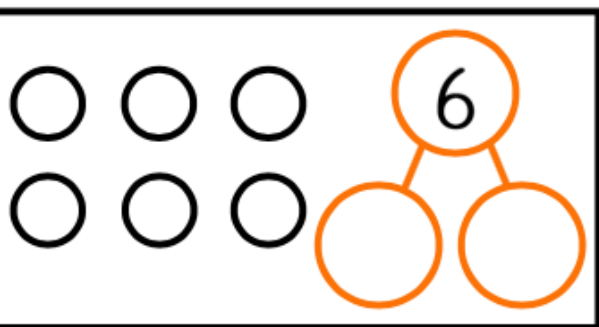
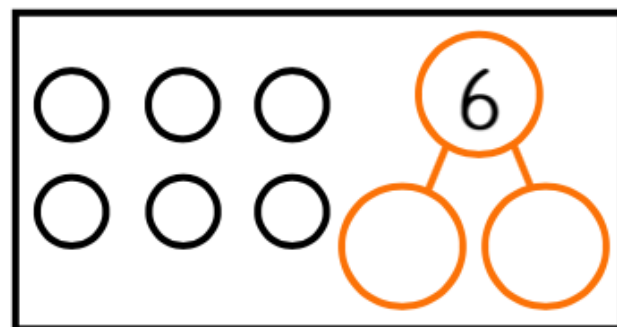
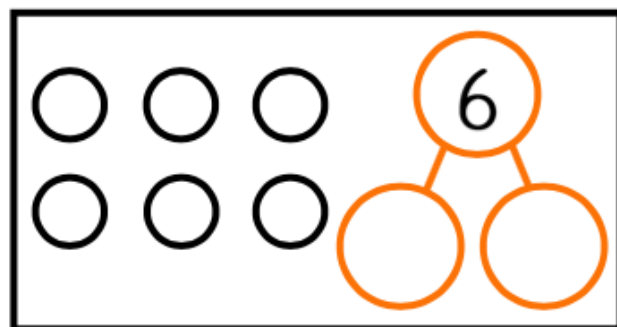
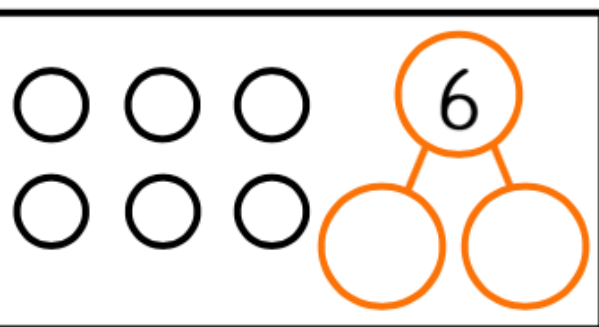


2. Colour dice number



3. Colour other part

4. The parts are...



Easiest? Hardest?

$$32 - 29$$

$$32 - 21$$

$$32 - 19$$

Different ways

$$26 - 18 = \square$$

Take away 20
then add \square

Count on from \square
to \square

Do 26 take away 16
then take away \square

Different ways

$$25 - 19 = \square$$

Take away \square
then add \square

Count on from \square
to \square

Do 25 take away 15
then take away \square

Digit cards game

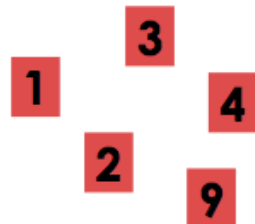
You need these digit cards:

Use each digit once.

Complete the number sentence.

$$\square \square - \square = \square \square$$

Challenge: do in different ways.



Digit cards game

You need digit cards 0 to 9

Use seven of the cards.

Complete the number sentences.



$$\square \square - \square = \square$$

$$\square - \square = \square$$

Challenge: use the **0** card.

Fill the gaps

$$\boxed{1} \boxed{3} - \boxed{8} = \boxed{5}$$

Spot the pattern

$$\boxed{1} \boxed{} - \boxed{7} = \boxed{5}$$

$$\boxed{1} \boxed{} - \boxed{6} = \boxed{5}$$

$$\boxed{1} \boxed{} - \boxed{5} = \boxed{5}$$

Fill the gaps

$$\boxed{1} \boxed{4} - \boxed{8} = \boxed{6}$$

Spot the pattern

$$\boxed{1} \boxed{4} - \boxed{} = \boxed{7}$$

$$\boxed{1} \boxed{4} - \boxed{} = \boxed{8}$$

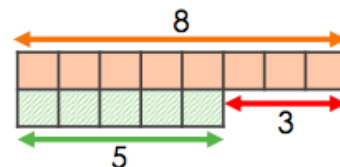
$$\boxed{1} \boxed{4} - \boxed{} = \boxed{9}$$

True or false? ✓ ✗

$$5 + 3 = 8 \quad \checkmark$$

$$8 - 3 = 5$$

$$8 - 5 = 3$$



$$3 + 5 = 8$$

$$5 - 8 = 3$$

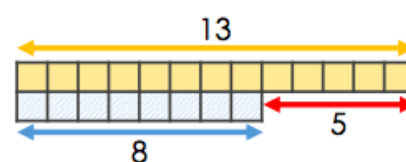
$$8 = 5 + 3$$

True or false? ✓ ✗

$$13 - 5 = 8 \quad \checkmark$$

$$5 - 13 = 8$$

$$13 - 8 = 5$$



$$5 = 8 + 13$$

$$13 = 5 + 8$$

$$13 - 5 = 8$$